1. Let $x, \varepsilon \in \mathbb{R}; \varepsilon > 0$. Take $\delta = \min(|\ln(f(x) - \varepsilon) - x|, |\ln(f(x) + \varepsilon) - x|)$. As the function is strictly increasing, for any y such that $|y - x| < \delta \iff x - \delta < y < x + \delta$ we will get $f(x) - \varepsilon < f(y) < f(x) + \varepsilon \iff |f(y) - f(x)| > \varepsilon$.

   Suppose f is uniform continous over $\mathbb{R}$. Then for a fixed $\varepsilon > 0$ there exists a δ such that for all $x, y \in \mathbb{R}$ such that $|x - y| < \delta$ we have $|f(x) - f(y)| < \varepsilon$.

   Take $k = \frac{\delta}{2}, y = x + k$. As $|x - y| < \delta$, we have $|f(y) - f(x)| < \varepsilon \iff e^x(e^k - 1) < \varepsilon$ for any x. But the left hand side is unbounded, so our assumption is false.

2. Let $f(x) = 0.5 \cos x$. Then $f^{-1}(a) = \cos^{-1}(2x)$.

   Then $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{-2}{\sin(\cos^{-1}(2x))} = \frac{-2}{\sqrt{1-4x^2}}$.

3. (a) $f'(x) = u'(x)v(x)w(x) + u(x)v'(x)w(x) + u(x)v(x)w'(x)$.
   $\frac{\delta g}{\delta x} = \frac{\delta u}{\delta x} vw + u \frac{\delta v}{\delta x} w + uv \frac{\delta w}{\delta x}$ (and similar for δy).

   (b) $\frac{\delta g}{\delta x} = 2x \sin(x - 3y)e^{xy} + (x^2 + y^2)\cos(x - 3y)e^{xy} + (x^2 + y^2)\sin(x - 3y)ye^{xy}$.
   $\frac{\delta g}{\delta y} = 2y \sin(x - 3y)e^{xy} - (x^2 + y^2)3\cos(x - 3y)e^{xy} + (x^2 + y^2)\sin(x - 3y)xe^{xy}$.

4. (a) $f'(x) = u'(v(w(x)))v'(w(x))w(x)$.
   $\frac{\delta g}{\delta x} = \frac{\delta u}{\delta v}\frac{\delta v}{\delta w}\frac{\delta w}{\delta x}$ and similarly for δy.

   (b) $\frac{\delta g}{\delta x} = -\cos(\cos(3x^5y^2 + 5x^3 + xy^3))\sin(3x^5y^2 + 5x^3 + xy^3)(15x^4y^2 + 15x^2 + y^3)$
   $\frac{\delta g}{\delta y} = -\cos(\cos(3x^5y^2 + 5x^3 + xy^3))\sin(3x^5y^2 + 5x^3 + xy^3)(6x^5y + 3xy^2)$

5. (a) $f(\pi + h) \approx f(\pi) + hf'(\pi) + \frac{h^2}{2}f''(\pi) + \frac{h^3}{6}f'''(\pi) + \frac{h^4}{24}f^{(4)}(\pi)x = h - \frac{h^3}{6}$.

   (b) Error term is $\frac{h^5}{120}f^{(5)}(\pi + \eta h) = -\frac{h^5}{120}\cos(\eta h)$, whose absolute value is at most $\frac{h^5}{120}$. On the given interval $(-\pi < h < \pi)$, this is at most $\frac{\pi^5}{120}$.

6. (a) Let $f(x) = e^{4x}$.
   $y \approx f(0) + xf'(0) + \frac{x^2}{2}f''(0) = 1 + 4x + 8x^2$.

   (b) $\lim_{s\to 0} \frac{e^{4s}-1}{s} = \lim_{s\to 0} \frac{-1+1+4s+8s^2+\ldots}{s} = \lim_{s\to 0} 4 + 8s + \ldots = 4$.

   (c) $(e^{4x})' = \lim_{s\to 0} \frac{f(x+s)-f(x)}{s} = e^{4x} \lim_{s\to 0} \frac{f(s)-f(0)}{s} = 4e^{4x}$

7. (a) $f'(x) = 6x^2 + 2x - 4$ and $f''(x) = 12x + 2$.
   Critical points are -1 and $\frac{2}{3}$. $f''(-1) = -10$ so -1 is a maximum, $f''(\frac{2}{3}) > 0$ so $\frac{2}{3}$ is a minimum.

   (b) $f'(x) = e^{x^3}3x^2$ and $f''(x) = 3((e^{x^3})'x^2 + e^{x^3}2x) = 3xe^{x^3}(3x^3 + 2)$.
   Critical points are $x = 0$. $f''(0) = 0$.
   $f''(-\varepsilon) = -\varepsilon 3e^{-\varepsilon^3}(2 - 3\varepsilon^3) < 0$ while $f''(\varepsilon) > 0$ so 0 is a saddle point.

8.

9. $\nabla f = \binom{4y\cos(4xy)}{4x\cos(4xy)}$ so the rate of change is $(\cos t, 1)\cdot\nabla f = 4t\cos t\cos(4t\sin t) + 4\sin t\cos(4t\sin t)$.

10. $\frac{\delta^2 u}{\delta y^2} = \left(\sin\theta\frac{\delta}{\delta r} + \frac{\cos\theta}{r}\frac{\delta}{\delta\theta}\right)\left(\sin\theta\frac{\delta u}{\delta r} + \frac{\cos\theta}{r}\frac{\delta u}{\delta\theta}\right) = \sin^2\theta\frac{\delta^2 u}{\delta r^2} - \frac{2\cos\theta\sin\theta}{r^2}\frac{\delta u}{\delta\theta} + \frac{2\cos\theta\sin\theta}{r}\frac{\delta^2 u}{\delta\theta\delta r} -$ $\frac{\cos^2\theta}{r}\frac{\delta u}{\delta r} + \frac{cos^2\theta}{r^2}\frac{\delta^2 u}{\delta\theta^2}$.

Adding the two equalities we get $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = \frac{\sin^2\theta+\cos^2\theta}{r}\frac{\delta u}{\delta r} + (\sin^2\theta + \cos^2\theta)\frac{\delta^2 u}{\delta r^2} + \frac{\sin^2\theta+\cos^2\theta}{r^2}\frac{\delta^2 u}{\delta\theta^2}$.

11. $\frac{\delta f}{\delta x} = 4x^3 + 8xy^2 - 4x$

$\frac{\delta f}{\delta y} = 8x^2 y + 4y$

So we have $x(x^2+2y^2-1) = y(2x^2+1) = 0$. So $x = y = 0$ or $x^2+2y^2 = 2x^2 = 1 \iff x = \sqrt{2}y$ and $y = 0.5$

$A_{2,0} = 2(3x^2+2y^2-1), A_{2,1} = 16xy, A_{2,2} = 2(2x^2+1)$

For $x = y = 0$ we get $A_{2,0} = -2, A_{2,1} = 0, A_{2,2} = 2$ which is a saddle point.

For $x = \frac{\sqrt{2}}{2}$ and $y = \frac{1}{2}$ we get $A_{2,0} = 2, A_{2,1} = 4\sqrt{2}, A_{2,2} = 4$ which is a maximum..

12. (a) $\frac{\delta T}{\delta t} = \frac{\delta T}{\delta\eta}\frac{\delta\eta}{\delta t} = f'(\eta)\frac{x}{\sqrt{D}}(-0.5)t^{-1.5}$

$\frac{\delta T}{\delta x} = f'(\eta)\frac{1}{\sqrt{Dt}}$

$\frac{\delta^2 T}{\delta x^2} = \frac{1}{Dt}f''(\eta)$

(b) We have $f'(n)\frac{x}{\sqrt{D}}(-0.5)t^{-0.5} = f''(\eta)$

Therefore $f''(\eta) + \frac{1}{2}\eta f'(n) = f'(n)\frac{x}{\sqrt{D}}(-0.5)t^{-0.5} + \frac{1}{2}\eta f'(n) = 0$.