Discrete Mathematics (Sheet #1)

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1. (a) {}
   (b) {-2, 0, 1}
   (c) {0, 1, 2}
   (d) all prime numbers
2. (a) $m + n$ and $\max(m, n)$
   (b) $\min(m, n)$ and 0
   (c) m, $\max(m - n, 0)$
   (d) $m + n$, 0
   (e) mn, mn
   (f) 2^m, 2^m
3. Commutativity, associativity, distributivity of $\cap$.
4. Suppose $A \subseteq B \wedge A \subseteq C$. Let $x \in A$. Then,

   $$x \in A \implies x \in B \wedge x \in C \implies x \in B \cap C$$

   Therefore all elements in A are also in $B \cap C$, so $A \subseteq B \cap C$.
   Now suppose $A \subseteq B \cap C$. Obviously $B \cap C \subseteq B$ and $B \cap C \subseteq C$.
   Since $\subseteq$ is transitive, $A \subseteq B \cap C \implies A \subseteq B \wedge A \subseteq C$.
   The second statement is false. A counterexample:

   $$\{1\} \subset \{1, 2\} \wedge \{1\} \subset \{1, 3\}, \text{ but } \{1\} \not\subset \{1, 2\} \cap \{1, 3\}$$

5. The first statement is true, the second is false.
   (a) $X \in \mathcal{P}(A \cap B) \iff X \subseteq A \cap B \iff X \subseteq A \wedge X \subseteq B \iff X \in \mathcal{P}(A) \wedge X \in \mathcal{P}(B) \iff X \in \mathcal{P}(A) \cap \mathcal{P}(B)$
   (b) Counterexample:

   $$\mathcal{P}(\{1\} \times \{2\}) \neq \mathcal{P}(\{1\}) \times \mathcal{P}(\{2\})$$

   The LHS is a set of sets, the RHS is a set of ordered pairs. Therefore they can't be equal unless both sets are empty, and in this case they aren't.
6. $A \setminus ((C \cap A) \cup B) = (A \setminus (C \cap A)) \cap (A \setminus B) = ((A \setminus C) \cup (A \setminus A)) \cap (A \setminus B) = ((A \setminus C) \cup \varnothing) \cap (A \setminus B) = (A \setminus C) \cap (A \setminus B) = A \setminus (B \cap C)$
7. The mistake is that $x \notin B \cap C$ does NOT imply $x \notin B \wedge x \notin C$.
   Counterexample: $\{1, 2\} \setminus (\{1\} \cap \{2\}) \not\subseteq (\{1, 2\} \setminus \{1\}) \cap (\{1, 2\} \setminus \{2\})$.
   LHS is $\{1, 2\}$ while RHS is $\varnothing$