7. (a) $\wedge$-intro: If we know x and y then we know $x \wedge y$.

$\vee$-elim: If we know $x \vee y$, we can show z assuming x and we can show z assuming y then we know z.

$\rightarrow$-intro: If we have a proof of y assuming x then we know $x \rightarrow y$.

Boxes provide scope for assumptions: An assumption made in a box is only available inside the box.

(b)

1.	$P \vee (Q \wedge R)$	premiss
2.	P	assumption
3.	$P \vee Q$	$\vee$-i
4.	$P \vee R$	$\vee$-i
5.	$(P \vee Q) \wedge (P \vee R)$	$\wedge$-i
6.	$Q \wedge R$	assumption
7.	Q	$\wedge$-e
8.	R	$\wedge$-e
9.	$P \vee Q$	$\vee$-i
10.	$P \vee R$	$\vee$-i
11.	$(P \vee Q) \wedge (P \vee R)$	$\wedge$-i
12.	$(P \vee Q) \wedge (P \vee R)$	$\vee$-i

Different proof? Huh? Maybe go backwards (finish with $\wedge$-i)?

(c) Reduction ad absurdum means that if we can prove a contradiction assuming ϕ then we know $\neg\phi$.

The proof rule is very similar to $\neg$-i: $\dfrac{[\neg\phi \ldots \bot]}{\phi}$.

(d) i.

1.	$\neg(P \vee Q)$	premiss
2.	P	assumption
3.	$P \vee Q$	$\vee$-i
4.	$\bot$	$\neg$-e
5.	$\neg P$	$\neg$-i
6.	Q	assumption
7.	$P \vee Q$	$\vee$-i
8.	$\bot$	$\neg$-e
9.	$\neg Q$	$\neg$-i
10.	$(\neg P) \wedge (\neg Q)$	$\wedge$-i

ii.

1.	$\neg(P \wedge Q)$	premiss
2.	$\neg(\neg P \vee \neg Q)$	assumption
3.	$(\neg\neg P) \wedge (\neg\neg Q)$	part (i)
4.	$\neg\neg P$	$\wedge$-e
5.	P	$\neg\neg$-e
6.	$\neg\neg Q$	$\wedge$-e
7.	Q	$\neg\neg$-e
8.	$P \wedge Q$	$\wedge$-i
9.	$\bot$	$\neg$-e
10.	$\neg P \vee \neg Q$	contradiction

8. (a) i.

1.	$\exists x \cdot R(x)$	premiss
2.	$\forall y \cdot \neg R(y)$	assumption
3.	$R(v)$	assumption; fresh v
4.	$\neg R(v)$	$\forall$-e
5.	$\bot$	$\neg$-e
6.	$\bot$	$\exists$-e
7.	$\neg\forall y \cdot \neg R(y)$	$\neg$-i

ii.

1.	$\neg\exists x \cdot \neg R(x)$	premiss
2.	fresh v	
3.	$\neg R(y)$	assumption
4.	$\exists x \cdot \neg R(x)$	$\exists$-i
5.	$\bot$	$\neg$-e
6.	$R(y)$	contradiction (classic)
7.	$\forall y \cdot R(y)$	$\forall$-i

iii.

1.	$\exists x \cdot R(x)$	premiss
2.	$\forall y \cdot \neg R(y)$	assumption
3.	$R(v)$	assumption; fresh v
4.	$\neg R(v)$	$\forall$-e
5.	$\bot$	$\neg$-e
6.	$\bot$	$\exists$-e
7.	$\neg\forall y \cdot \neg R(y)$	$\neg$-i

(b) i. =-i: $\dfrac{}{T = T}$

=-e: $\dfrac{A = B, \phi(A)}{\phi(B)}$

ii.

1.	$T_1 = T_2$	assumption
2.	$T_1 = T_1$	=-i
3.	$T_2 = T_1$	=-e

iii.

1.	$\forall x \cdot (x = 0 \lor x = 1)$	premiss
2.	$\exists x \cdot R(x)$	premiss
3.	$\neg R(1)$	premiss
4.	$R(v)$	assumption; fresh v
5.	$v = 0 \lor v = 1$	$\forall$-e
6.	$v = 0$	assumption
7.	$R(0)$	=-e
8.	$v = 1$	assumption
9.	$R(1)$	=-e
10.	$\bot$	$\neg$-e
11.	$R(0)$	$\bot$-e
12.	$R(0)$	$\lor$-e
13.	$R(0)$	$\exists$-e